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Short Communication

AN ANALYTICAL METHOD FOR COMPUTATION OF INDUCTION MACHINE INDUCTANCES UNDER ROTOR MISALIGNMENT FAULT

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ABSTRACT

A great deal of research has so far been conducted on the analytical computation of induction machine inductances under different eccentricity conditions. However, they consider the radial non-uniformity and neglect the non-uniformity in the axial direction, but in practice the axial non-uniformity due to rotor misalignment (RM) fault is quite common. This paper presents an analytical model of three phases squirrel cage induction machine under RM fault. The proposed method is able to calculate the time varying inductances versus rotor angle for three phase squirrel cage induction machines under different RM conditions. Simulation results are compared to those obtained from ordinary method and experiments.

Keywords: Analytical model, Rotor misalignment, Inductance calculation, Induction machine.

INTRODUCTION

A big innovation in modeling and simulation of AC motors, based on winding function approach (WFA), has been introduced by Toliyat *et al.* (1991). This method has been shown to be accurate for the simulation and analysis of AC motor performance based on the basic geometry and winding layout of machine. The only information required in WFA is the machine geometry and winding layout (Toliyat and Lipo, 1995). The calculation quickness of this approach constitutes its main advantage when several machine non-uniformities are to be studied. An essential part of this approach is the calculation of machine inductances. Contrary to the classical d-q model, this method can implicitly take into account all space harmonics. Moreover, by this approach, stator and rotor asymmetries as well as a non-uniform air gap can be taken into account. Hence, this method has been widely and satisfactorily used for the analysis of asymmetrical and fault conditions in electrical machines, such as broken rotor bars (Toliyat and Lipo, 1995; Milimonfared *et al.*, 1999) and fault condition in stator windings (Joksomovic and Penman, 2000).

An important modification of winding function approach and a new method to calculate inductances under nonsymmetrical air-gap, called the modified winding function approach (MWFA) was presented by Al-Nuim and Toliyat (1998). MWFA is indeed very well suited to analyze induction machines with air gaps of arbitrary shape. Although, modern methods based on FEM give accurate results, however these methods are time consuming especially for the analysis of electrical machines with asymmetry in the motor body such as air gap eccentricity (DeBortoli *et al.*, 1993; Gyftakis and Kappatou, 2013). In eccentricity condition, the rotor axis is separated from the stator axis, while the rotor axis reminds parallel to that of the stator. Inclined air gap eccentricity or rotor misalignment (RM) occurs when the rotor axis is not parallel to the stator axis. Then, the air gap changes in both axial and radial directions. In reality, the most probable case is the RM fault. Different eccentricities in induction machines are documented previously (Joksimovic *et al.*, 2000; Nandi *et al.*, 2002; Faiz *et al.*, 2003; Meshgin *et al.*, 2003; Joksimovic, 2005; Faiz and Ojaghi, 2009; Ghoggal *et al.*, 2012). These models are not able to analyze the effects produced by the axial air-gap non-uniformity in induction machines.

An extension of the MWFA for the inductance calculation considering axial non-uniformity was proposed by Bossio *et al.* (2004). This method was used to calculate the mutual inductance of a three phase induction machine under axial non-uniformity condition. In recent years, different RM faults in induction machines have been studied (Ediadong and Nandi, 2007; Dorrell, 2009;

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Akbari *et al*., 2010; Akbari, 2013; Kaikaa and Hadjami, 2014). These techniques use numerical integration or inexact analytical equations based on approximated Fourier series expansions of the inverse air gap function for calculation of inductances under axial non-uniformity. This paper develops a precise analytical expression for induction machine inductances under RM condition. This will ensure that all space harmonics ignored by the Fourier series expansions of the inverse air gap function are included in the model. The proposed technique is used to calculate the inductances of a three-phase squirrel cage induction machine under different RM conditions. Effects of several rotor axial asymmetries on inductances in these conditions are shown. For dynamic simulation and study of line current spectra, calculated inductances are used in a coupled electromagnetic model. Line current spectra from proposed method are compared to those obtained from ordinary MWFA. Finally, inductance profiles from simulation and experiment are compared to verify the proposed model. Comparison between simulation and experimental results indicates a good agreement.

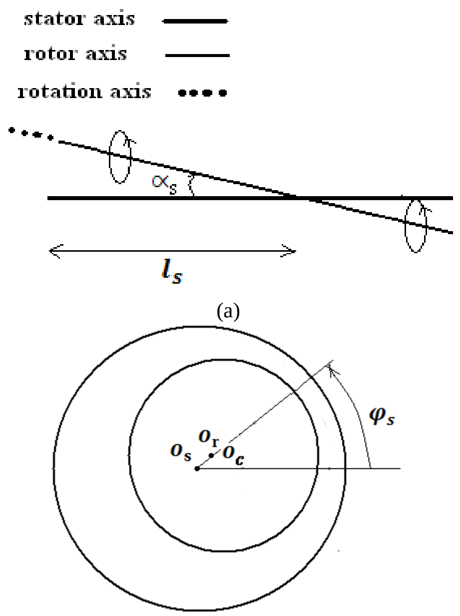


Fig. 1. RM fault in an induction machine. (a) axes position and (b) a cross section of machine. o_s is stator center, o_r is rotor center and o_c is rotor rotating center.

MODELING RM FAULT IN INDUCTION MACHINE

RM is caused by improper alignment of right and left bearing centers. Consequently, along the axial direction, the degree of eccentricity is not constant. To model the RM, this fault can be treated as a variable circumferential eccentricity.

Figure 1 shows an elementary induction machine with RM fault. In this condition, the symmetrical axis of rotor, which is superimposed to the rotor rotating axis, is inclined relative to the symmetrical axis of stator. The air-gap function changes in non-uniformity conditions compared to the symmetrical case. In this condition, the air gap length variation can be described by air gap function as follows:

$$g(\varphi,\varphi_S,z)=g_0(1-\delta(z)cos(\varphi-\varphi_s)) \tag{1}$$

where g_0 is effective air gap length in healthy condition, g is air gap function in case of RM, φ_s is angle at which rotor rotation and stator centers are separated and δ is eccentricity level. The stator and rotor slots effect is included by Carter's coefficient referred to the slots (Nandi, 2004). φ_s is clearly shown in figure 1-b. δ is obtained from the ratio between the distance of stator and rotation centers ($o_s o_c$ in Fig. 1-b) and the air gap length of healthy machine. Through geometric analysis on figure 1-a, it is straight forward to show that the eccentricity level at any point along the axial direction as

$$\delta(z)=-tan(\alpha_s)(z-l_s)/g_0 \tag{2}$$

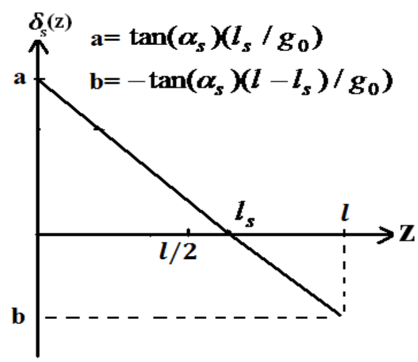


Fig. 2. Variation of eccentricity coefficient in different positions for machine under RM fault.

α_s is the inclined angle of the rotor and l_s is the rotor shaft misalignment level. α_s and l_s are clearly shown in Fig. 1-a. The variation of air gap eccentricity coefficient versus axial position in this condition is shown in figure 2.

ANALYTICAL EXPRESSION FOR INDUXTANCES UNDER RM FAULT

Inductances of electrical machines can be calculated by the winding function theory. The general expression for mutual inductance between any two windings *a* and *b* in any electrical machine, considering axial nonuniformity, is given by the following equation (Bossio *et al*., 2004).

$$L_{ab}=\mu_\circ r\int_0^l\int_0^{2\pi}n_a.M_b.g^{-1}d\phi dz \tag{3}$$

where

$$M_b = n_b - \frac{\int_0^l\int_0^{2\pi} g^{-1}.n_b d\varphi dz}{2\pi l\int_0^l\int_0^{2\pi} g^{-1} d\varphi dz} \quad (4)$$

$\mu_\circ$ is free space permeability, l is axial stack length of the machine, g^{-1} is inverse air gap function, r is mean radius of the air gap, n_a is turn function of winding *a* and M_b is modified winding function of winding *b*. The winding functions of the stator phases and rotor loops of the machine do not change under the air gap non-uniformity conditions compared to the symmetrical condition.

Calculation of inductances by (3) requires that precise equations for the inverse air gap function be obtained. Generally, in calculating induction machine inductances under axial non-uniformity by (3), numerical integration or inexact analytical equations based on approximated Fourier series expansions of the inverse air gap function are used. To enhance the accuracy of calculated inductances, more terms of Fourier series expansion must be included. This makes the computation more complex.

In this investigation, analytical expression for inductances of induction machine under RM fault are derived without any development in Fourier series. To develop an analytical expression, it is assumed that the function f_g is indefinite integral of the function g^{-1} as follows:

$$f_g(\varphi,\varphi_s,z) = \int g^{-1}(\varphi,\varphi_s,z)d\varphi = \int \frac{1}{g_0(1-\delta(z)cos(\phi-\phi_s))} d\phi \quad (5)$$

This expression is elaborated to yield

$$f_g(\phi,\phi_s,z) = \frac{1}{g_0\sqrt{1-\delta(z)^2}} cos^{-1}\left(\frac{cos(\phi-\phi_s)-\delta(z)}{1-\delta(z)cos(\phi-\phi_s)}\right) \quad (6)$$

It is assumed that the functions f_1 and f_2 to be the definite integrals as follows:

$$f_1(\varphi_s) = \int_0^l\int_0^{2\pi} g^{-1}(\varphi,\varphi_s,z)d\varphi dz = \int_0^l\left[\int_0^{2\pi} g^{-1}(\varphi,\varphi_s,z)d\varphi\right]dz \quad (7)$$

$$f_2(\varphi_s) = \int_0^l\int_0^{2\pi} g^{-1}(\varphi,\varphi_s,z)n_b(\varphi,z)d\varphi dz = \int_0^l\left[\int_0^{2\pi}(g^{-1}(\varphi,\varphi_s,z))n_b(\varphi,z)d\varphi\right]dz \quad (8)$$

Using the definition given in (5), f_1 can be obtained.

$$f_1(\varphi_s) = \int_0^l\left[f_g(2\pi,\varphi_s,z) - f_g(0,\varphi_s,z)\right]dz \quad (9)$$

To calculate the f_1, the function f_1' is defined as

$$f_1' = f_g(2\pi,\phi_s,z) - f_g(0,\phi_s,z) \quad (10)$$

Therefore

$$f_1(\phi_s) = \int_0^l f_1'(\phi_s,z)dz \quad (11)$$

Dividing the machine axially to *m* cross sections and using the trapezoidal rule for integration, f_1 is obtained as follows

$$f_1(\phi_s) = \sum_{j=1}^m \int_{\frac{l(j-1)}{m}}^{\frac{lj}{m}} f_1'(\phi_s,z)dz = \frac{l}{m}\sum_{j=1}^m f_1'(\phi_s,z_j) \quad (12)$$

Where

$$z_j = \frac{\frac{l(j-1)}{m}+\frac{lj}{m}}{2} = \frac{1}{2m}(2lj-l) \quad (13)$$

In a similar way, f_2 is calculated in the whole range of φ. Function f is defined as follows:

$$f(\varphi_s) = \frac{f_2(\varphi_s)}{2\pi l f_1(\varphi_s)} \quad (14)$$

Using the definitions given in (7), (8) and (14), MWF of stator phase *b*, M_b, is derived as

$$M_b(\phi,\phi_s,z) = n_b(\phi,z) - \frac{\int_0^l\int_0^{2\pi} g^{-1}(\phi,\phi_s,z)n_b(\phi,z)d\phi dz}{2\pi l\int_0^l\int_0^{2\pi} g^{-1}(\phi,\phi_s,z)d\phi dz} = n_b(\phi,z) - \frac{f_2(\phi_s)}{2\pi l f_1(\phi_s)} = n_b(\phi,z) - f(\phi_s) \quad (15)$$

To develop an expression for L_{ab}, L'_{ab} is defined as

$$L'_{ab} = \int_0^{2\pi} n_a(\varphi,z)M_b(\varphi,\varphi_s,z)g^{-1}(\varphi,\varphi_s,z)d\varphi \quad (16)$$

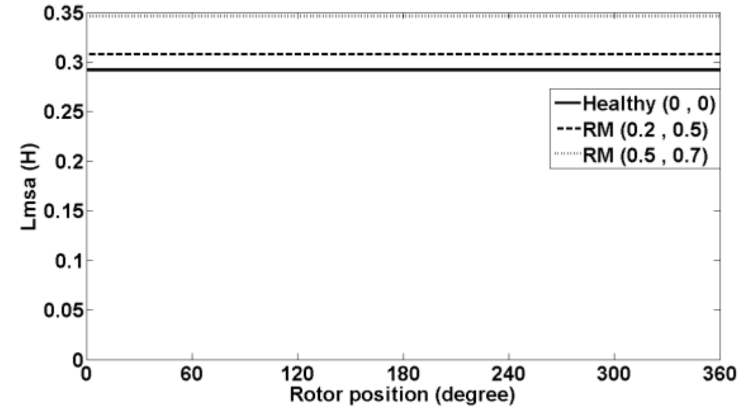
Therefore

$$L_{ab} = \mu_\circ r\int_0^l\int_0^{2\pi} n_a(\phi,z)M_b(\phi,\phi_s,z).\, g^{-1}(\phi,\phi_s,z)d\phi dz = \mu_\circ r\int_0^l[\int_0^{2\pi} n_a(\phi,z)M_b(\phi,\phi_s,z).\, g^{-1}(\phi,\phi_s,z)d\phi]dz$$

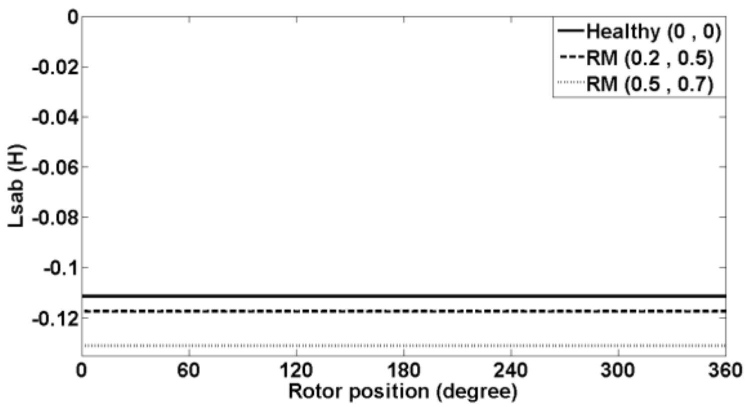
$$= \mu_\circ r \int_0^l \left[L'_{ab}(\phi_s, z) \right] dz \tag{17}$$

Using the trapezoidal rule for integration, L_{ab} is obtained as follows:

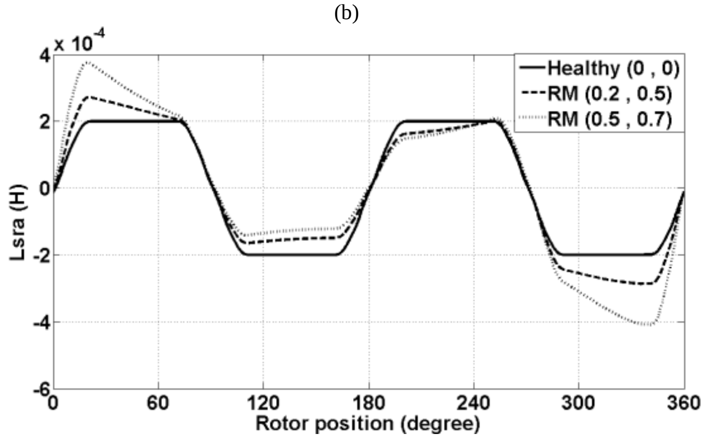
$$\begin{aligned} L_{ab} &= \mu_\circ r \int_0^l \left[L'_{ab}(\phi_s, z) \right] dz \\ &= \mu_\circ r \sum_{j=1}^{m} \int_{\frac{l(j-1)}{m}}^{\frac{lj}{m}} L'_{ab}(\phi_s, z) dz \\ &= \frac{\mu_\circ l r}{m} \sum_{j=1}^{m} L'_{ab}(\phi_s, z_j) \end{aligned} \tag{18}$$



(a)



(b)



(c)

Fig. 3. Magnetizing inductance of stator phase A (a), mutual inductance between phase A and phase B (b) and mutual inductance between phase A and rotor loop1 (c), as a function of rotor position under healthy and RM conditions.

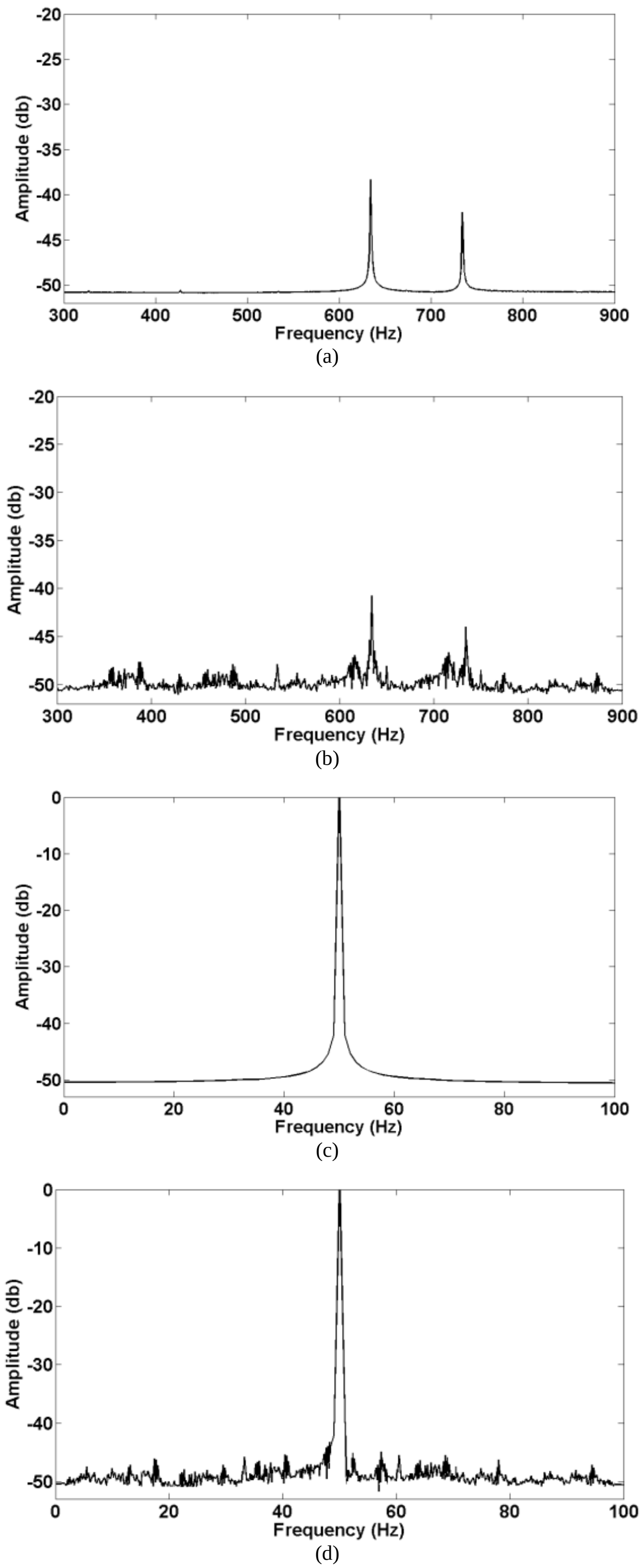


Fig. 4. Simulated normalized stator current spectra, around PSH and supply frequency for the healthy machine, using the proposed technique (a) and (c) and ordinary MWFA (b) and (d).

Fig. 5. Induction motor with moveable bearings.

To calculate the L'_{ab}, it is assumed that, φ_i is angle of stator slot *i* center, φ_{ti} is angle of stator tooth *i* (angular position between stator slot *i* and stator slot *i+1*). Using the definitions given in (5) and (15), L'_{ab} can be obtained.

$$L'_{ab} = \int_0^{2\pi} n_a(\varphi,z) M_b(\varphi,\varphi_s,z) g^{-1}(\varphi,\varphi_s,z) d\varphi$$
$$= \sum_{i=1}^{n} [n_a(\phi_{ti},z)(n_b(\phi_{ti},z) - f(\phi_s)).$$
$$\int_{\phi_i}^{\phi_{i+1}} g^{-1}(\phi,\phi_s,z) d\phi]$$
$$= \sum_{i=1}^{n} [n_a(\phi_{ti},z)(n_b(\phi_{ti},z) - f(\phi_s)).$$
$$(f_g(\phi_{i+1},\phi_s,z) - f_g(\phi_i,\phi_s,z))] \qquad (19)$$

where n is number of stator slots and functions f and f_g are calculated from (14) and (6), respectively.

The inductances of induction machines can therefore be calculated analytically by means of (18) without any development in Fourier series. It is clear that the proposed technique leads to more accurate results. Moreover, the time and computation process decrease. This technique has capability to simulate the mechanical asymmetry and fault of stator and rotor with no restrictions about the axial and radial non-uniformity.

CALCULATION OF INDUCTANCES

In this section, inductances of a three phase induction machine are computed for different RM conditions. Table 1 shows the specifications of the simulated motor.

Table 1. Specifications of simulated motor.

Rated power, hp	3
Rated voltage, V	380
Number of poles	4
Number of stator slots	36
Rated frequency, Hz	50
Number of rotor slots	28
Inner diameter of stator (mm)	90
Air gap length (mm)	0.3
Core length (mm)	90

All the inductances, obtained for different rotor angular positions, are stored within a computer file. It was assumed that the magnetic saturation and leakage flux are negligible. Figure 3 shows how the profile of the inductances change with RM. Eccentricity level at two ends of machine is specified on these plots. For example, In RM (0.2 , 0.5), eccentricity level at two ends of machine is 0.2 and 0.5. As shown in figure 3, RM causes an asymmetrical mutual inductance between stator phase A and rotor loop 1, a symmetrical magnetizing inductance of stator phase A and a symmetrical mutual inductance between stator phase A and phase B. The magnetizing inductance of stator phase A and mutual inductance between stator phase A and phase B are independent of rotor position. Its reason is that the air gap reluctance is rotor position independent. By increasing the average eccentricity level, the magnitude of inductances increases. For example, the value of magnetizing inductance of the healthy machine, 0.29 H increases to 0.312 H in the case of DM (0.2, 0.5) and to 0.345 H in the case of DM (0.5, 0.7).

For studying the frequency spectra of stator line current, the calculated inductances were used in a coupled electromagnetic model (Al-Nuim and Toliyat, 1998). Fig.

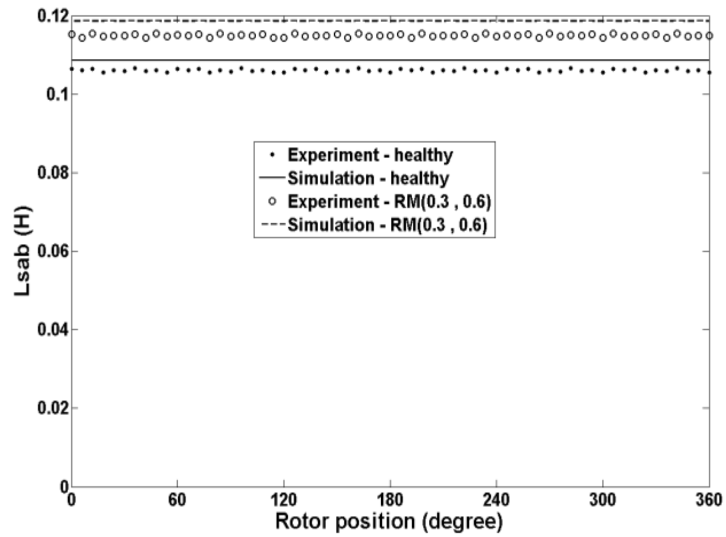


Fig. 6. The magnitude of mutual inductance between stator phase A and phase B under healthy and RM (0.3, 0.6) conditions for the experiment and simulation

4 shows the normalized frequency spectra of the line current for the healthy machine obtained with the proposed technique and ordinary MWFA. As shown in these plots, the noise level of the stator line current spectra from the proposed method is lower than the MWFA results. Therefore, with the proposed method, it is possible to study the behavior of more weak frequency component. The reason is that the developed analytical expression prevents the imprecision caused by numerical differentiations.

RESULTS AND DISCUSSION

In order to validate the simulation results, a three phase induction motor, similar to the simulated one, was tested under healthy and RM conditions. A digital oscilloscope was used to measure the voltage and current. To be able to impose misalignment on the rotor, a special test bed as used. As shown in figure 5, the stator and the rotor bearings at the two ends of the motor were separately mounted on the test bed. By this manner, the induction machine with different degrees of RM can be provided for tests.

To obtain the mutual inductance between stator phase A and phase B, a sinusoidal voltage was applied in stator phase A and the voltage induced in phase B was measured for different rotor positions from 0^o to 360^o taking 6^o steps. The mutual inductance was calculated as follows:

$$L_{AB}(\theta) = \frac{V_B(\theta)}{V_A(\theta)} L_{AA}(\theta) \qquad (20)$$

where $V_A(\theta)$ and $V_B(\theta)$ are induced voltages in stator phase A and phase B, respectively and $L_{AA}(\theta)$ is self inductance of stator phase *A*, previously calculated by means of phase *A* voltage and current measurements.

Figure 6 shows the mutual inductance between stator phase *A* and phase *B* under healthy and RM conditions for the experiment and simulation. As shown, the experimentally obtained inductance profiles are very similar to those obtained using the proposed technique.

The discrepancy between results from the proposed method and the experiments is due to neglecting the magnetic saturation and leakage flux. The author plans to extend the method in the future to consider these effects.

CONCLUSION

In this study, a more precise analytical expression for induction machine inductances under RM condition has been developed. The proposed method is able to analytically calculate the inductances of induction machine with no restrictions about the axial and radial non-uniformity. The proposed method was used to calculate the inductances of the induction machine, under healthy and different RM faults. Good agreement was found between the simulated and the experimental inductances profiles. Comparison between the results obtained by presented method and those obtained by previous method based on MWFA shows that the noise level of the line current spectra from proposed method is lower than the MWFA results. Using the developed analytical expression, the imprecision due to numerical differentiations is eliminated. The behavior of more weak frequency components can therefore be studied by the proposed method. Calculation of inductances is an essential step for simulation and analysis of fault in induction machines, therefore the proposed method will improve the on-line diagnosis of RM fault.

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